



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 35

Forward Pricing – Consumption Assets

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CASH & CARRY ARBITRAGE

- Transaction costs, costs of carrying the asset are ignored to keep the exposition tractable.
- F_0 is the arbitrage free forward price.
- F^*_0 is the actual forward price.
- Let $F^*_0 > F_0 = S_0 \exp(rT)$.



- Consider the following strategy:
- (t=0)
- Borrow S_0
- Buy asset and hold the asset for (0,T)
- Take a short forward position.
- (t=T)
- Deliver the asset against short forward position,
- Receive cash F^*_0 ,
- Pay off loan with interest $S_0 \exp(rT)$.
- Since $F^*_0 > F_0 = S_0 \exp(rT)$ you end up with positive cash.

REVERSE CASH & CARRY ARBITRAGE

- Let $F^*_0 < F_0 = S_0 \exp(rT)$.
- Consider the following strategy:
- (t=0) Borrow the asset S and sell for S_0 , invest this amount at r% for (0,T). Take a long forward position.
- (t=T) Receive invested amount with interest $S_0 \exp(rT)$. Pay F^*_0 against forward and receive the asset, restore it to owner.
- Since $F^*_0 < S_0 \exp(rT)$ you end up with positive cash.

C&C AND REVERSE C&C WITH DIVIDENDS & CARRYING COSTS

- Cash & Carry arbitrage will occur when:
- $F^*_0 > F_0 = (S_0 - D_0 + U_0) \exp(rT)$
- Reverse Cash & Carry arbitrage will occur when:
- $F^*_0 < F_0 = (S_0 - D_0 + U_0) \exp(rT)$

EXAMPLE 1: FORWARD PRICING

- A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is 40 and the risk-free rate of interest is 10% par annum with continuous compounding. At that time the forward price is 46. Is there any possibility of arbitrage, if so, how can arbitrage profit be made?



SOLUTION						
GIVEN INFORMATION						
SPOT PRICE			S(0)	40	UNITS	
RISKFREE RATE			r	10	PERCENT	
TENURE			T	1	YEAR	
THEREFORE,						
NO ARBITRAGE FORWARD PRICE		$S(0)\exp(rT)$		44.2068		
ACTUAL FORWARD PRICE				46		
HENCE ARBITRAGE IS POSSIBLE						

EXAMPLE 2: FORWARD PRICING

- On 1st April 2021, a stock was expected to pay a dividend of 2.10 per share in two months ($t=2$) and in five months ($t=5$). The stock price at this date ($t=0$) was 50, and the risk-free rate of interest was 24% p.a. with continuous compounding for all maturities. Calculate the no-arbitrage forward price for a 6-month forward contract on the stock.

SOLUTION					
TIME		0.0000	2.0000	5.0000	6.0000
STOCK PRICE		50.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.1000	0.1200
DISCOUNT FACTOR			-0.0400	-0.1000	-0.1200
			0.9608	0.9048	
DIVIDEND			2.1000	2.1000	
PV OF DIVIDEND			2.0177	1.9002	
		3.9178			
NET STOCK PRICE AT T=0	$X(0)=S(0)-D(0)$	46.0822			
FVIF					1.1275
FORWARD PRICE AT T=6	$X(0)*FVIF$	51.9575			

REVERSE ARBITRAGE: $F^*_0 < F_0 = (S_0 - D_0 + U_0) \exp(rT)$ CONSUMPTION ASSETS

Reverse arbitrage occurs when you buy forward and sell spot.

Reverse arbitrage may not be used for a commodity that is a consumption asset rather than an investment asset.

Individuals who own a consumption commodity usually plan to use it in some way. They are reluctant to sell/lend the commodity in the spot market and buy forward, because forward and futures contracts cannot be consumed.

Hence, reverse arbitrage may not operate so that $F^*_0 < F_0 = (S_0 - D_0 + U_0) \exp(rT)$ may hold for such assets.

CONVENIENCE YIELD

- For consumption assets, reverse C&C arbitrage may not operate. As a result $F^*_0 < (S_0 - D_0 + U_0) e^{rT}$ may hold.
- Thus, we can write $F^*_0 = (S_0 - D_0 + U_0) e^{(r-y)T}$.
- The factor y i.e. the convenience yield measures the extent to which the forward price is lower than the future value of spot price. Thus, y is indicative of preference of the consumers to own and retain the asset and consume it rather than have a forward contract on it.

- The convenience yield reflects the market's expectations concerning the future availability of the commodity. The greater the possibility that shortages will occur, the higher the convenience yield.
- Again, if users of the commodity have high inventories, there is very little chance of shortages in the near future and the convenience yield tends to be low. If inventories are low, shortages are more likely and the convenience yield is usually higher.

EXAMPLE 3: CONVENIENCE YIELD

- The current price of wheat is Rs 1,200 per quintal. Storage costs work out to Rs 150 per quintal per annum payable quarterly in advance. Calculate the convenience yield (in % per annum) if the forward price per quintal of wheat is 1,425 for a forward contract with maturity of one year. The risk free interest rate is given as 10% p.a continuously compounded.

SOLUTION						
GIVEN INFORMATION						
SPOT PRICE			S(0)	1200	PER QTL	
RISKFREE RATE			r	10	PERCENT	
TENURE			T	1	YEAR	
STORAGE COSTS			U	150	PER YEAR	
PAYABLE QUARTERLY IN ADVANCE						
PRESENT VALUE OF STORAGE COSTS						
QUARTER	AMOUNT	PVIF	PV			
1	37.5	1	37.5			
2	37.5	0.9753099	36.57			
3	37.5	0.9512294	35.67			
4	37.5	0.9277435	34.79	144.536		
NO ARBITRAGE FORWARD PRICE				1485.94		
ACTUAL FORWARD PRICE				1425		
EXP (yT)				1.04277		
y				4.19%		

VALUING A FORWARD AFTER INCEPTION

- Consider a long position in a forward contract A set up at $t=0$ with maturity T and price $F(0,T)$.
- Let, at a later time t^* , a forward contract B on the same underlying for maturity at T have a forward price of $F(t^*,T)$.
- Then, both contracts A & B entail delivery of the same asset at same time ($t=T$) but at different prices $F(0,T)$ and $F(t^*,T)$.

- Clearly, if $F(0,T) < F(t^*,T)$, a long position in the original forward A will command a positive value.
- Further, this value will be the present value the difference $F(t^*,T) - F(0,T)$ so that
- $V_A(t^*,T) = [F(t^*,T) - F(0,T)]e^{-r(T-t^*)} = S_{t^*} - F(0,T)e^{-r(T-t^*)} = (S_{t^*} - S_0 e^{rt^*})$.



$t_0 = \text{JAN 1}, F_0 = 75$
LONG

$t^* = \text{APR 1}, F_{t^*} = 90$
SHORT

$T = \text{DEC 31},$
 $CF = 90 - 75 = 15$

$$V_{\text{portfolio}} = V_A - V_B = V_A = \text{PV of 15 at } t=t^* = \text{PV}(F_{t^*,T} - F_{0,T})$$

The value is positive and riskfree since 15 shall be received with certainty at $t=T$.